Meeting 7

Math 22

Activity 10.2.5

- Finish parts (c)-(e) as a class.

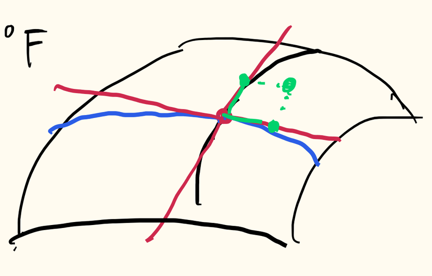
(a) $w_v(20,-10) = -0.5$ Hint: $f(x+h) \approx f(x) + hf'(x)$

(b) $w_T(20,-10) = 1.3$

(c) $w(18,-10) = w(20-2,-10) \approx w(20,-10) + (-2)w_v(20,-10)$
$= -35 + 1 = -34°F$

(d) $w(20,-12) \approx w(20,-10) + (-2)w_T(20,-10)$
$= -35 - 2.6 = -37.6°F$

(e) $w(18,-12) = w(20,-10) + (-2)w_T(20,-10) + (-2)w_v(20,-10)$
$= -35 - 2.6 + 1 = -36.6°F$



Activity 10.2.6

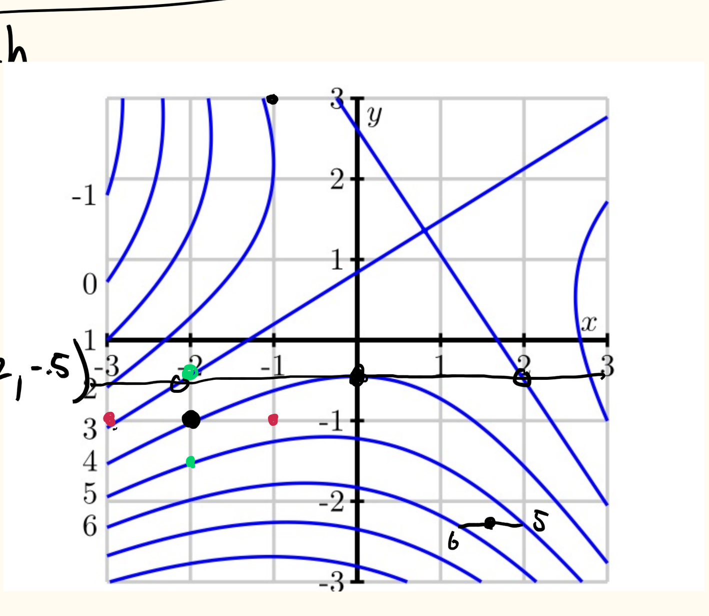
- Complete as a class $f'(a) \approx \dfrac{f(a+h) - f(a-h)}{2h}$

(a) $f_x(-2,-1) \approx \dfrac{f(-2+h,-1) - f(-2-h,-1)}{2h}$

(h=1)
$= \dfrac{f(-1,-1) - f(-3,-1)}{2}$

$= \dfrac{4.5 - 3}{2} = \dfrac{3}{4}$

(b) $f_y(-2,-1) = -f(-2,-1.5) + f(-2,-.5)$
$= -5 + 3$
$= -2$



(c) Skip

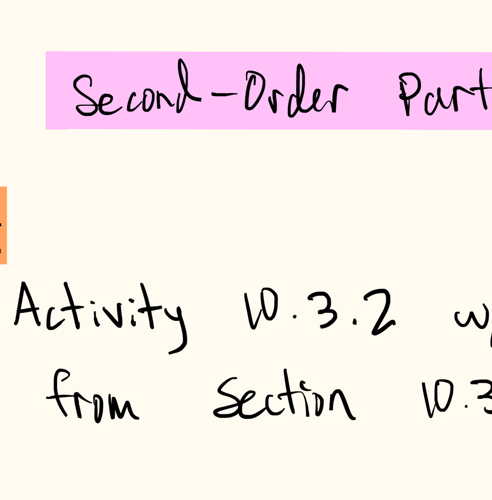
(d) One point (x,y) where $f_x(x,y) = 0$
$f_x(0,-.5) = 0$

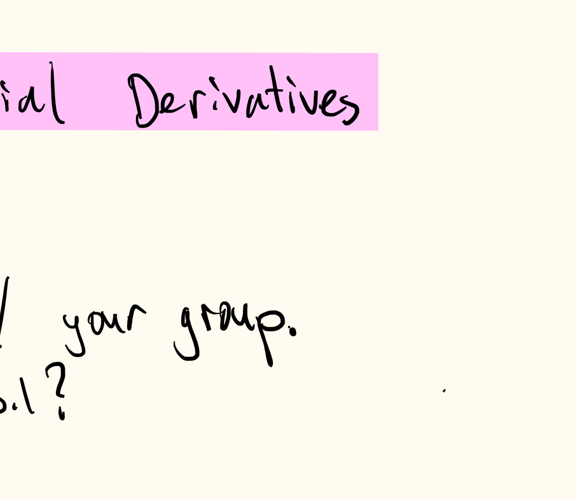
(e) $f_x(x,y) < 0$ $f_x(1.5,-2.25) < 0$

(f) $f_y(x,y) > 0$ $f_y(-1,3) > 0$

(g) $g(2,2) = 4$
$g_x(2,2) > 0$
$g_y(2,2) > 0$

(h) $h(2,2) = 4$
$h_x(2,2) < 0$
$h_y(2,2) > 0$





Section 10.3 Second-Order Partial Derivatives

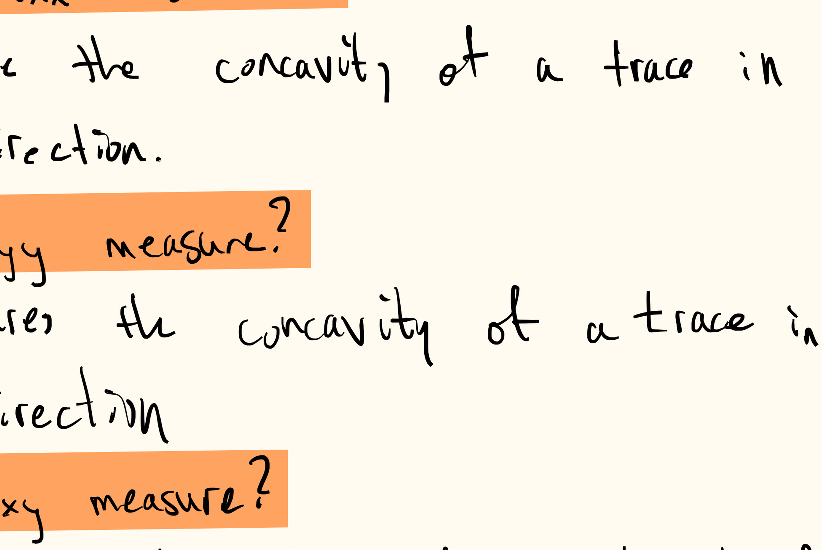
Reading Debrief

- Discuss Activity 10.3.2 w/ your group.
- Questions from Section 10.3.1?

Section 10.3.2 Interpreting Second-Order Partials

Recall: what does $f''(x)$ measure?

- $f'(x)$ = slope of tangent line
- $f''(x)$ measures concavity of the graph of



$f''(a) < 0$ $f''(a) = 0$, $f''(x) > 0$
$x < a$ $x > a$

What does f_{xx} measure?

- f_{xx} measure the concavity of a trace in the x-direction.

What does f_{yy} measure?

- f_{yy} measures the concavity of a trace in the y-direction

What does f_{xy} measure?

- f_x measures the slope of the tangent line to a trace in the x-direction
- f_{xy} measures the rate of change of the slope in the x-direction as we move in the y-direction. Measures the "twisting" of the graph as we move along a trace in the y-direction.

Activity 10.3.4

- Complete w/ your group.
- Class discussion. $f'(a) \approx \dfrac{f(a+h) - f(a-h)}{2h}$

(a) $w_T(20,-15) = \dfrac{w(20,-10) - w(20,-20)}{10} = \dfrac{-35 - (-48)}{10} = 1.3$

$w_T(20,-10) = \dfrac{w(20,-5) - w(20,-15)}{10} = \dfrac{-29 - (-42)}{10} = 1.3$

$w_T(20,-5) = \dfrac{w(20,0) - w(20,-10)}{10} = \dfrac{-22 - (-35)}{10} = 1.3$

$w_{TT}(20,-10) = \dfrac{w_T(20,-5) - w(20,-15)}{10} = 0.$

(c) $w_T(25,-10) = \dfrac{w(25,-5) - w(25,-15)}{10} = \dfrac{-31 - (-44)}{10} = 1.3$

$w_T(15,-10) = \dfrac{w(15,-5) - w(15,-15)}{10} = \dfrac{-26 - (-39)}{10} \doteq 1.3$

$w_{Tv}(20,-10) = \dfrac{w_T(25,-10) - w_T(15,-10)}{10} = \dfrac{1.3 - 1.3}{10} = 0$

Section 10.4 Tangent Planes and Differentials

Reading Debrief

- Discuss Activity 10.4.2 w/ your group.
- Questions from Section 10.4.1?

Tangent Plane If $f(x,y)$ is continuously diff. at (x_0,y_0), then the tangent plane to the graph of f at (x_0,y_0) exists and has scalar equation:

$$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0).$$

Q: What information about f can be read from the tangent plane at (x_0,y_0).

A: You can read the values $f_x(x_0,y_0)$, $f_y(x_0,y_0)$, $f(x_0,y_0)$.

Q: Suppose $2x-3y+2z=7$ is tangent to f at $(1,-1)$. What are

A: $f(1,-1) = 1$
$f_x(1,-1) = -1$
$f_y(1,-1) = \frac{3}{2}$

$L(x,y) = z = \frac{7}{2} - x + \frac{3}{2}y$

Q: Does every function have a tangent plane at every point?

A: No. The function $f(x,y) = x^{1/3}y^{1/3}$ has no tangent plane at $(0,0)$. See GeoGebra.

Section 10.4.2 Linearization

The linearization of a function $f(x,y)$ at a point (x_0,y_0) is the function $L(x,y)$ whose graph is the tangent plane to f at (x_0,y_0):

$$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

The function $L(x,y)$ provides a "good" approximation of f when (x,y) is near (x_0,y_0).

Activity 10.4.3

- Complete Activity 10.4.3 and discuss w/ your group.
- Class discussion.

(a) $g(1,2) = \frac{1}{5}$ $g_x(x,y) = \dfrac{x^2+y^2-2x^2}{(x^2+y^2)^2} = \dfrac{y^2-x^2}{(x^2+y^2)^2}$

$g_x(1,2) = \dfrac{3}{25}$

$g_y(x,y) = \dfrac{-2xy}{(x^2+y^2)^2}$ $g_y(1,2) = \dfrac{-4}{25}$

$L(x,y) = \frac{1}{5} + \frac{3}{25}(x-1) - \frac{4}{25}(y-2)$

$g(.8,2.3) \approx L(.8,2.3) = .1288$